§ 2.8: One-to-One Functions and Their Inverses

One-to-One Functions

Definition of a One-to-One Function

A function f with domain A is called a **one-to-one function** if no two elements of A have the same image, that is,

$$f(x_1) \neq f(x_2) \qquad \text{whenever} \qquad x_1 \neq x\_2.$$

Horizontal Line Test

A function is one-to-one if and only if no horizontal line intersects its graph more than once.

Example 1	Deciding whether a Function is One-to-One

Is the function $f(x) = x^3$ one-to-one?

Example 2	Deciding whether a Function is One-to-One

Is the function $f(x) = x^2$ one-to-one?

Example 3	Deciding whether a Function is One-to-One

Is the function $f(x) = x^2$ with domain $[0, \infty)$ one-to-one?

Example 4	Showing that a Function if One-to-One

Show that the function $f(x) = 4x + 7$ is one-to-one.

The Inverse of a Function

Definition of the Inverse of a Function

Let f be a one-to-one function with domain A and range B. Then its **inverse function**, denoted by f^{-1}, has domain B and range A and is defined by

$$f^{-1}(y) = x \quad \Leftrightarrow \quad f(x) = y$$

for any y in B.

Example 5	Finding f^{-1} for Specific Values

Let $f(1) = 5, f(3) = 7,$ and $f(8) = \pi$. Find $f^{-1}(7), f^{-1}(\pi),$ and $f^{-1}(5)$.

Inverse Function Property

Let f be a one-to-one function with domain A and range B. The inverse function f^{-1} satisfies the following cancellation properties

$$f^{-1}(f(x)) = x \qquad \text{for every } x \text{ in } A$$

$$f(f^{-1}(x)) = x \qquad \text{for every } x \text{ in } B$$

Conversely, any function f^{-1} which satisfies these equations is the inverse of f.

Example 6	Verifying That two Functions are Inverses

Show that $f(x) = x^3$ and $g(x) = x^{1/3}$ are inverses.

How to Find the Inverse of a One-to-One Function

1. Write $y = f(x)$.
2. Solve this equation for x in terms of y (if possible).
3. Interchange x and y. The resulting equation is $y = f^{-1}(x)$.

Example 7	Finding the Inverse of a Function

Find the inverse of the function $f(x) = 2x + 7$, then check your work.

Example 8	Deciding whether a Function is One-to-One

Find the inverse of the function $f(x) = \frac{x^7 - 3}{5}$, then check your work.

The graph of f^{-1} is obtained by reflecting the graph of f in the line $y = x$.

Example 9	Finding the Inverse of a Function

(a) Sketch the graph of $f(x) = \sqrt{x+2}$.

(b) Use the graph of f to sketch the graph of f^{-1}.

(c) Find an equation for f^{-1}.

Homework

Due:_____

2 – 54 (even)